International Journal of Scientific Research and Management (IJSRM)
||Volume||07||Issue||02||Pages||EC-2019-238-240||2019||
Website: www.ijsrm.in ISSN (e): 2321-3418
Index Copernicus value (2015): 57.47, (2016):93.67, DOI: 10.18535/ijsrm/v7i2.ec04

Pin-ended column buckling load is 2.73% lower than Euler's

Abdulqader S. Al-Najmi[1]

Department of Civil Engineering, School of Engineering, the University of Jordan. P.O.Box 13298, Amman 11942, Queen Rania Street, Amman-JORDAN.,

Abstract

A verification calculation was aimed at further proving that the least critical load gives the Euler buckling load for a pin-ended column to be: $P_{cr}=\frac{\pi^2 EI}{L^2}$. Selecting a parabolic function for this purpose, that meets the boundary conditions and is almost exactly similar to the symmetrical function sin(x). It was expected to obtain a critical load that is larger than the load given by the sine function. The parabolic function produced a lower critical load by about 3%.

Keywords: neutral buckled shape, critical load, conjugate beam, elastic weights.

Introduction:

Figure 1 gives the neutral buckled shape of a pin ended compression member, free to rotate about frictionless pins at its ends.

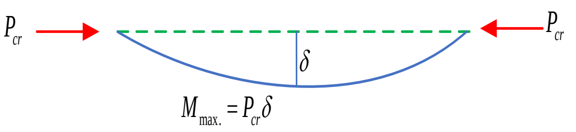


Figure 1: Neutral buckled shape of a pin ended column displaced parabolically

When the axial load P is less than P_{cr}, the compression member remains straight and undergoes only axial compression deformation. When the load is increased gradually, a condition of neutral equilibrium is reached. At this stage of loading, the column theoretically may have any infinitesimally small lateral deflection initiated by a very small lateral force, and upon removing this lateral force, the lateral deflected shape does not disappear. The buckled shape is possible only at a critical or Euler load, as prior to this load the column remains straight. The smallest load at which a buckled shape is possible is the critical load. The compression member will be at condition of neutral equilibrium that is described by the equation:

$$\frac{M}{EI}=\frac{d^2v}{dx^2}=-P_{cr}v$$

by letting $\lambda^2=\frac{P_{cr}}{EI}$, and rearranging, gives

$$\frac{d^2v}{dx^2}+\lambda^2 v=0$$

The solution of the homogeneous second order linear differential equation that adheres to meet the boundary conditions is: $P_{cr}=\frac{\pi^2 EI}{L^2}$

A comeback on the Analysis of pin ended column

The solution of the homogeneous second order linear differential equation benefits from the use of the exponential function, which can be represented by trigonometric functions for the case of complex roots of the characteristic equation (auxiliary equation). The case of pin ended columns requires a symmetric function to observe the boundary conditions, hence the analysis is restricted to the trigonometric sine function: $v = A\sin\lambda x$

From which the boundary condition:

$v=0$ at $x=L$ from which $\lambda L = n\pi$ or $\lambda^2 L^2 = n^2\pi^2 \rightarrow P_{cr} = \dfrac{\pi^2 EI}{L^2}$ It can be seen that although the solution is founded on the exponential function and its equivalent form in terms of trigonometric functions, the boundary conditions forces the symmetrical shape on the solution in any case. A parabolic function fits these requirements, it is symmetrical, and its derivatives are symmetric. The use of such function does not mean that it shall bring about a smaller critical load, but it did. The suggested parabolic function produced a lower critical load by about 2.73%.

Assume the deflected shape at the neutral equilibrium is a parabola of the second degree as shown in Figure 2. The bending moment diagram is the parabolic shape multiplied by P_{cr}, and the elastic weights are obtained by dividing the moments by EI.

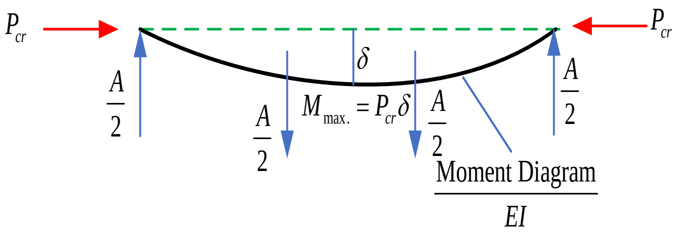


Figure 2: Conjugate beam showing the elastic weights Considered in calculations

Area of the parabola $= \dfrac{2}{3}\dfrac{P_{cr}\delta L}{EI} = A =$ elastic weight of the bending moment diagram.

Figure 2 shows the conjugate beam with the relevant elastic weights.

Deflection at the middle of the span:

$$\delta = \frac{A}{2}\times\left[\frac{L}{2}-\frac{3}{8}\times\frac{L}{2}\right] = \frac{5}{48}P_{cr}\delta\times\frac{L^2}{EI}$$

From which:

$$P_{cr} = \frac{48}{5}\times\frac{EI}{L^2} = \frac{97.27}{100}\times\frac{\pi^2 EI}{L^2}$$

The other case that admits a similar solution is the fixed cantilever with an axial load as shown in Figure 3:

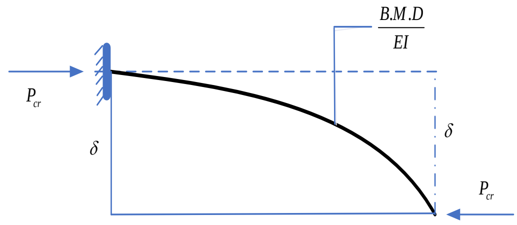


Figure 3: A Cantilevered column displayed at its neutral buckled shape

In this case, the maximum moment will be $P_{cr}\times\delta$. From the properties of the parabolic shape of the bending moment diagram, the conjugate beam shown in Figure 4 is used to calculate the deflection at the tip.

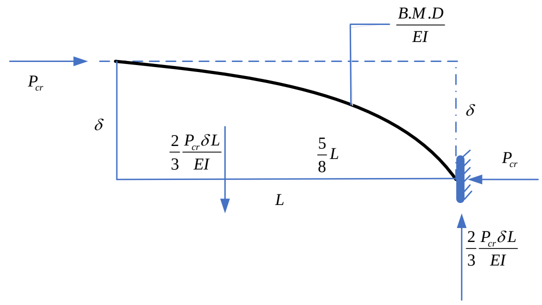


Figure 4: Conjugate beam of the buckled column with elastic loads used calculations

The elastic weight of the parabolic bending moment diagram $= \dfrac{2}{3}\dfrac{P_{cr}\delta L}{EI}$

$$\delta = \frac{2}{3}\frac{P_{cr}\times\delta\times L}{EI}\times\frac{5}{8}L$$

$$= \frac{2.4\times EI}{L^2} = \frac{97.27}{100}\times\frac{\pi^2}{4}\times\frac{EI}{L^2}$$

In the Appendix, A general derivation is attached.

Conclusions:

1. The pin ended column buckles at a load smaller than Euler's buckling load by 2.73%. The same result applies to the cantilevered column.
2. The other cases of Euler's buckling loads for different boundary conditions, may not be immune from any reduction.

Notation

E = Modulus of elasticity in tension and compression

I	=	Moment of inertia of cross-sectional area
L	=	Length of column
M	=	Bending moment
P	=	Axial force
P_{cr}	=	Euler buckling load
r	=	Radius of gyration
v	=	Transverse deflection of beam
δ	=	Maximum deflection of the neutrally buckled beam
λ	=	Eigen value in column buckling problems

References

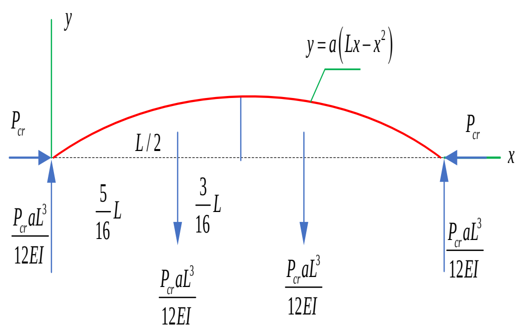
[1] Erwin Kreyszig, (2006), Advanced Engineering Mathematics, 9th Edition, John Wiley and sons, Inc., Chapter two.

[2] Popov, Egor, (1978), Mechanics of Materials, 2nd Edition, Prentice-Hall International Editions, Chapter 13.

[3] Timoshenko, S. (1958), Strength of Materials, part II, Advanced Theory and Problems, 3rd Edition, Van Nostrand Reinhold, Chapter V.

Author Profile: Abdulqader **S.** Al-Najmi received B.Sc. degree in Civil Engineering from Cairo University in 1972, his M.Sc. and Ph.D. degrees from Manchester University in 1977 and 1980 respectively. Currently, Professor of Civil Engineering at the University of Jordan.

Appendix



Appendix Figure: Neutral parabolic displaced shape

Parabolic equation $y=a\left(Lx-x^2\right)$ … (1)

$$\text{at } x=\frac{L}{2}\rightarrow y=\frac{aL^2}{4}.$$

The bending moment at any section distanced x is:

$$M_x=-P_{cr}a\left(Lx-x^2\right) \quad \ldots (2)$$

Dividing by EI

$$\frac{M_x}{EI}=-\frac{P_{cr}a\left(Lx-x^2\right)}{EI} \quad \ldots (3)$$

Equation 3 can be written as follows:

$\frac{d^2y}{dx^2}+\frac{P_{cr}}{EI}y=0$, which satisfies the original differential equation.

$A=$ Area of half the parabola (considering elastic weights)

$$A=\int_0^{L/2}\frac{P_{cr}a\left(Lx-x^2\right)}{EI}dx$$

$$A=\frac{P_{cr}aL^3}{12EI}$$

$\bar{x}=$ centroid of half the area of the parabola from the end

$$A\times\bar{x}=\int_0^{L/2}\frac{M_x}{EI}x\,dx$$

$$\frac{P_{cr}aL^3}{12EI}\bar{x}=\int_0^{L/2}\frac{P_{cr}a\left(Lx-x^2\right)x\,dx}{EI}$$

$$=\frac{5P_{cr}aL^4}{192EI}\Rightarrow\bar{x}=\frac{5}{16}L$$

$$\delta=\frac{aL^2}{4}=\frac{P_{cr}aL^3}{12EI}\left(\frac{L}{2}-\frac{3L}{16}\right)\rightarrow P_{cr}=\frac{48}{5}\frac{EI}{L^2}$$