

# Optimizing Spillway Capacity by Considering the Possibility of Flood Forecasting

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## Abstract:

As one of the most important and expensive hydraulic structures, dams are typically built in a multi-planar form, with flood control being one of their primary goals. Consequently, a specific portion of the reservoir volume is designated for flood control in dam construction projects. For safety reasons, this section must always be kept empty to prevent spillway in the event of a flood. If it is feasible to reduce or remove this volume without compromising the dam's safety, it can result in a significant reduction in costs. With the advancement of automatic flood warning systems, predicting the hydrograph and the timing of flood occurrences in the basin has become possible. This information can be utilized for dams equipped with such systems. Thus, in basins prone to flooding, it is possible to receive advance notice (pre-warning time) about the timing and volume of the impending flood. Consequently, a portion of the reservoir can be emptied in anticipation, creating a dedicated volume for flood control without the need to allocate a separate section of the reservoir or reducing the initially planned volume. An alternative perspective is that by implementing this strategy, a portion of the reservoir's volume serves dual purposes acting as water storage during normal times and as flood control volume during floods. This dual functionality contributes to a decrease in project costs, enhancing the economic justification for numerous projects. In this study, we investigated the method to achieve this objective and developed software that utilizes the aforementioned strategy to design spillway and determine the most economical option.

**Keywords:** Spillway capacity, optimization, flood warning system, trend finding.

## 1. Introduction

While humanity has coexisted with floods throughout history, the devastating impacts of floods today surpass those of the past. It is evident that it is no longer feasible to entirely prevent all floods; instead, the focus is on minimizing the damage through effective management. Flood management methods are broadly categorized into two groups: structural methods and non-structural methods. Structural methods possess three key characteristics – they are implemented pre-emptively, predominantly entail physical structures, and are designed to divert floods away from populated areas (Li et al., 2010; Smith & Simpson, 2011). Any approach addressing flood-related issues but lacking these three characteristics is classified as a non-structural method in flood management. Among the structural methods, the construction of dams along flood paths stands out. However, due to the high construction costs involved, dams are typically built with a multi-purpose approach. Consequently, only a portion of the reservoir volume is dedicated to flood control (Castillo et al., 2007; Karaboga et al., 2008; Sanders et al., 2006).

One non-structural method with a long history in flood management is flood warning systems. Over time, these systems have been widely accepted as an effective tool to reduce flood damage through the application of science and technology. Given the high costs associated with structural flood management systems, there is a current trend towards integrating structural methods with non-structural ones (Evrard et al., 2008; Hasebe & Nagayama, 2002). The objective of this study is to integrate two flood damage reduction methods—dams and flood warning systems—during the design stage. The aim is to enhance safety while concurrently reducing the overall costs by utilizing the services of the non-structural component (Carrivick et al., 2011; Gallegos et al., 2009).

## 2. . Determining the base flood of the spillway plan

One of the crucial parameters considered in spillway design is the selection of the design flood, typically achieved through two methods: a) using established criteria, and b) employing the optimal risk method. Generally, the first method is applied in designs. The rules implemented in this approach encompass general instructions that each country compiles based on plan requirements, cultural-social factors, economic considerations, safety aspects, etc., forming the foundation for the plan. In the optimal risk method, a total cost function is formulated, aiming to minimize the risk, which is the combined cost of spillway and downstream damage. The determination and design of the spillway are then carried out based on this established risk limit (Chuntian & Chau, 2002; Dai et al., 2005).

**Table 1:** Variations of optimal early warning time with flood return period for different damage coefficients

<i>Flood Return period</i>	<i>Optimal pre-warning time</i> ( $D=4$ )	<i>Optimal pre-warning time</i> ( $D=10$ )	<i>Optimal pre-warning time</i> ( $D=20$ )
1000	0	0	0
2000	0	0	0
5000	60	0	0
1000	18	36	0

## 3. . Optimum spillway design considering the flood warning system

In areas where the utilization of a warning system is feasible, the individual in charge of the dam is responsible for announcing the likelihood of an impending event within the next hour (pre-warning time). This pre-warning time can be employed to create a flood control volume (T) by emptying an additional portion of the reservoir. This action contributes to greater relief of the flood within the tank, resulting in a reduction of the flood peak and ultimately lowering the maximum water level compared to scenarios without a warning system. Consequently, the dam's height can be decreased. Taking this into consideration, adjusting the warning time and modifying the threshold level and spillway width can lead to an option where the overall costs of the system are minimized (Lind et al., 2009; O'Connor et al., 2003). The subsequent article will explain the methods employed to achieve this objective.

The spillway investigated in this study is a peak spillway equipped with a valve, situated outside the dam structure. This system allows for a probability ( $q$ ) of accurate flood warnings - $q$  hours prior to the event in the area, thanks to the existence of an automatic flood warning system. The system enables the prediction of the hydrograph for the flood entering the reservoir. Given that the warning system is fully automated, the valves are fully opened following a warning (Poulard et al., 2010; Stunden Bower, 2010). The most critical scenario occurs when the reservoir's water level is at the normal level. In this state, the initial discharge from the spillway corresponds to the flow generated by the water head, equal to the difference between the normal water level and the threshold level. Kamyab Moghaddam et al., in their experimental study of energy loss in a stepped spillway, investigated that the energy loss is associated with the geometry of steps. Their results show that the incline of steps dramatically influences energy loss. Their approach has been followed in this research to optimize the size of the spillway and maximize the energy dissipation accordingly. (Kamyab Moghaddam et al., 2022). For optimization, the design flood is also initially selected, similar to conventional spillway designs. The selection is based on standards assuming zero risk. Consequently, this study will delve into explaining the optimal risk method, which encompasses the zero-risk state (Bhadra et al., 2008; Singh et al., 2011).

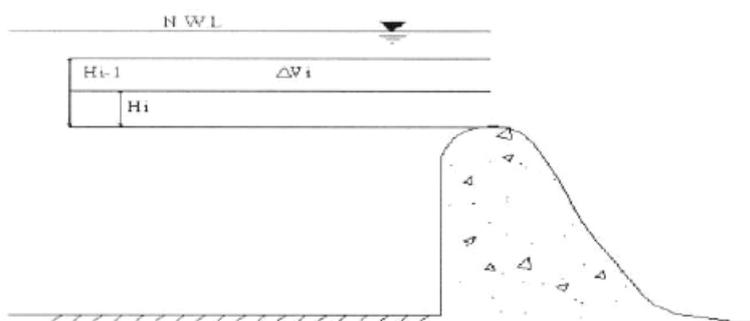
#### 4. . How to determine the maximum allowable spillway width considering the flood warning system

It is assumed that when the flood warning is issued, the spillway valves of the dam are fully opened, initiating the process of emptying the reservoir. Depending on the selected spillway width, three situations may arise: The reservoir level during the flood should be higher than the spillway threshold level but lower than the normal level. Simultaneously with the arrival of the flood at the reservoir, the reservoir level will also reach the level of the spillway threshold. Sometime before the flood reaches the reservoir, the reservoir level has already reached the spillway threshold level.

The additional control volume created by emptying the tank is the same in modes 2 and 3. The difference lies in the width of the spillway, which is wider in the third case than in the second case. Naturally, costs are higher in the third case. Therefore, the width of the spillway should not be so large that the water level reaches the spillway threshold before the flood reaches the tank (Hayashi et al., 2008; Xia et al., 2011).

Assuming that the volume of water between the normal level and the threshold level consists of small volumes  $\Delta v_i$  and denoting the water head corresponding to each level as  $H_t$  (Figure 1), the average time required to empty the water volume between the two levels is equal to  $H_i$ , given  $H_{i-1}$ :

$$\Delta t_i = \frac{\Delta v_i}{\left(\frac{q_i + q_{i-1}}{2}\right) * L} \quad (1)$$



**Figure 1:** Determine the allowable width of the spillway

The time required to empty the total water between the normal level and the level of the spillway threshold, represented as the sum of these small times  $\Sigma \Delta t_i$ , should, according to the above, be smaller than the flood warning time, expressed as  $(\Sigma \Delta t_i \leq T)$ . Through algebraic simplification, the maximum allowable width for the spillway is presented in the following relation:

$$L_{max} = \frac{2}{T} * \Sigma \frac{\Delta v_i}{(q_i + q_{i-1})} \quad (2)$$

where:  $\Delta t_i$  = partial discharge time,  $\Delta v_i$  = partial volumes between two levels  $i, i-1$ ,  $q_i$  = discharge of unit width with a water head equal to  $H_i$ ,  $L_{max}$  = maximum allowed spillway width,  $L$  = spillway width,  $T$  = pre-warning time.

#### 5. . How to determine the flood trend in the reservoir of a dam with a valved spillway assuming the existence of a pre-warning time

The pulse method, a hydrological approach used for reservoir trend tracking, has been employed for this purpose. As mentioned earlier, once the flood warning is received, the valves are fully opened. The most critical scenario occurs when the water level is normal. Consequently, the initial discharge from the spillway equals the discharge from the spillway with a water level corresponding to the normal level. This discharge

occurs within the time interval of the flood reaching the tank, equivalent to the pre-warning time. The incoming flood's flow rate is assumed to be zero (Clements, 2009; Kang et al., 2007). Applying the aforementioned assumptions to the pulse method, the water level of the reservoir at the moment of the flood and the water level at the time of the peak flood can be differentiated in five general situations:

1- Before the flood reaches the reservoir, reaching the threshold level of the water is not possible. This limitation is due to the maximum virtual width constraint discussed earlier (Hydrograph 1, Fig. 2).

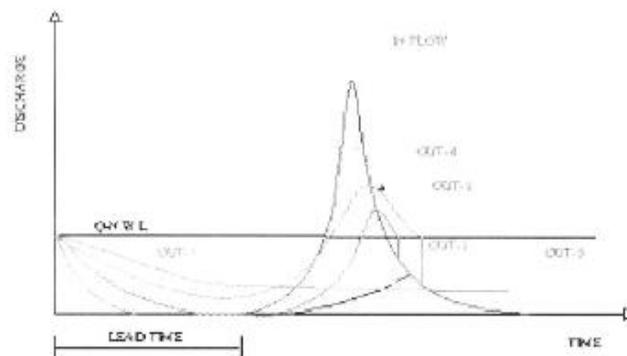
2- Simultaneously with the flood reaching the reservoir, the water level coincides with the spillway threshold level. When the flood passes through such a reservoir, its peak is reduced, and the water level at the time of the peak is either greater than or equal to the normal level (Hydrograph 2, Fig. 2).

3- At the moment the flood reaches the reservoir, the water level coincides with the spillway threshold level, and it attains the peak flood level during the flood process within the reservoir. However, this level is below the normal level. In other words, the peak flood flow is less than the flow discharged from the spillway with the water level equal to the normal level (Jasper et al., 2002; Kovács & Perrochet, 2008). This indicates that the spillway width is too large, making this option uneconomical (Hydrograph 3, Fig. 2).

4- In this case, when the flood reaches the reservoir, the water level is lower than both the spillway threshold level and the normal level, allowing the design flood to enter the tank (Hydrograph 4, Fig. 2).

5- The current follows a downward course, indicating a large volume of flood control and a high discharge equivalent to the normal level compared to the incoming peak flood (Hydrograph 5, Fig. 2).

During the optimization process, options 2 and 4 are included in the calculations, while options 1, 3, and 5 are excluded from consideration.



**Figure 2:** Possible types of hydrographs exiting the reservoir assuming the existence of a pre-warning time

## 6. . Cost function and how to calculate it

The ultimate selection criterion is the total cost of the options. The option with the lowest cost is chosen as the final design. Consequently, it is essential to calculate the total cost for each option, encompassing six costs: spillway cost, valve cost, dam cost, cost of damages caused by a flood larger than the design flood, and the cost of damages resulting from false warnings. We will explain each of these costs in detail:

### 6.1 Spillover cost (CS)

Spillage costs incorporate excavation expenses, including quarrying and earthmoving for channel walls, chutes, and the energy consumption structure (Salisbury & Hagen, 2007; Sehnert & Lindenschmidt, 2009). These costs are dependent on the width of the spillway and the threshold level. It's important to note that the impact of foundations is not considered in these calculations. However, this cost can be factored into the valve cost.

### 6.2 The cost of valves (CG)

The cost of the valve is determined by the dimensions of the valve. The weight of the valves is calculated using the method outlined in the reference.

$$G = k_t \cdot h^{0.63} \cdot W^{1.1} \cdot H \quad (3)$$

where  $h$  = valve height,  $w$  = valve width,  $H$  = water head behind the valve,  $G$  = valve weight,  $k_t$  = correction factor for the ratio of valve height to width. The cost of valves is obtained by multiplying the weight of one valve by the number of spillway valves in the unit price of valve weight.

### 6.3 Dam construction cost (CD)

In this study, the dam construction cost is considered as a function of the level of the dam crest. This level itself is determined by the maximum water level in the tank and the freeboard, both of which are influenced by the maximum flow rate of the flood and the level of the spillway threshold.

### 6.4 The cost of damage caused by a flood that is greater than the flood of the plan (CO)

In the event of a flood larger than the planned flood, the water passing over the crest of the dam will inevitably result in damage or destruction of the entire dam and downstream facilities. Taking into account the bank interest rate ( $\alpha\%$ ), the linear price growth rate ( $\beta\%$ ), and the cost of damage caused by the destruction of the dam in the first year as a multiplier of the initial construction cost of the dam, the total present value of the cost of damage caused by the destruction of the dam over the useful life of the project ( $T$  years) will be determined (Loukas, 2002; Yoshikawa et al., 2010)+.

$$CO = \sum_{t=1}^T (1-p) \cdot p^{t-1} q \cdot (1+\alpha)^{-t} (1+\beta)^t \lambda \cdot CD \quad (4)$$

where  $p$  = the probability of the design flood not occurring or greater than that during one year, the annual linear growth rate of the price =  $\beta$  the annual linear growth rate of the price,  $\alpha$  = the annual bank interest rate,  $\lambda$  = constant coefficient,  $T$  = life of the bridge,  $CD$  = cost the beginning of the construction of the dam,  $CO$  = the present value of the cost of damage caused by the failure of the dam.

### 6.5 Cost of damages caused by false warning (CFW)

There is a possibility of a design flood occurring every year after the construction of the dam. According to previous discussions and the laws of probability, the probability of its occurrence  $t$  years after the dam's construction is given by  $(1-p) p^{t-1}$ . If the probability of false warning is specified for the pre-warning time  $q$ , then the probability of the design flood occurring  $t$  years after the construction of the dam and a false warning being given will be equal to  $pt=(1-p)p^{t-1}q$ . Taking into account the bank interest rate ( $\alpha$ ) percent and the linear price growth coefficient ( $\beta$ ), and considering the cases mentioned in the previous topic, the present value of the costs of damages caused by water spillway due to false warning during the useful life of the dam ( $T$  years) is equal to.

$$CFW = \sum_{t=1}^T (1-p) \cdot p^{t-1} q \cdot (1+\alpha)^{-t} (1+\beta)^t \lambda \cdot CD \quad (5)$$

It should be mentioned that the probability of a false alarm is a function of the pre-alarm time. In other words, as the warning time increases, the probability of a false alarm also increases.

### 6.6 The cost of servicing the flood warning system (CPW)

This fee is paid periodically to the guardian of the warning system. Since the cost of servicing the alarm system for flooding is a fixed cost and has no relation to spillway capacity or the design flood return period, it will not impact the selection of the optimal option. For simplicity, this cost can be excluded from the total costs (CT).

$$CT = CS + CG + CD + CO + CFW + CPW \quad (6)$$

Therefore, based on the above, the total cost function (formula (6)) can be expressed in the following simplified form.

$$CT = CS + CG + \left[ 1 + \lambda \cdot (1 + q)(1 + \beta)(1 + \alpha)^{-1} \frac{[1 - P^T(1 + \beta)^T(1 + \alpha)^{-T}]}{[1 - P(1 + \beta)^T(1 + \alpha)^{-1}]} \right] \cdot CD \quad (7)$$

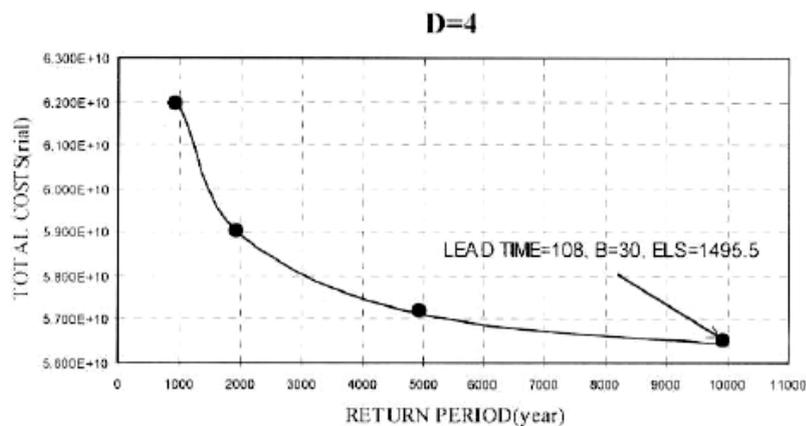
### 6.7 A Case Study

In this study, available information was utilized for the initial selection of an existing earthen dam with spillway situated outside the dam body. Key parameters include a normal water level of 1498 meters, a riverbed level at the dam site of 1460 meters, a free height of 2 meters, a useful dam life of 50 years, and an spillway coefficient of 1.9.

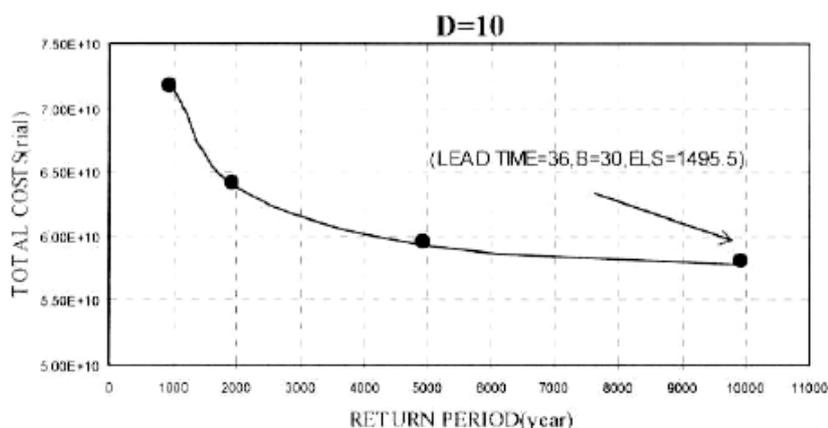
The investigation in this study considered three threshold levels (1495.5, 1493, 1490.5) and eleven spillway widths (30, 35, 40, 45, 75, 70, 65, 60, 55, 50, 80). Costs were calculated based on current prices of similar ongoing projects. Given the downstream facilities and limited information on damage caused by spillway, three damage factor (D) values (40, 10, and 20) were chosen. Design floods were selected with return periods of 1000, 2000, 5000, and 10000 years.

Considering the absence of an automatic flood warning system in the basin, a hypothetical system was assumed. Statistical studies on its past performance revealed a linear relationship between the available pre-warning time and the probability of receiving a false alarm, up to 132 hours of advance warning. For every 12 hours of increased pre-warning time, a 5% probability of receiving a false alarm was added.

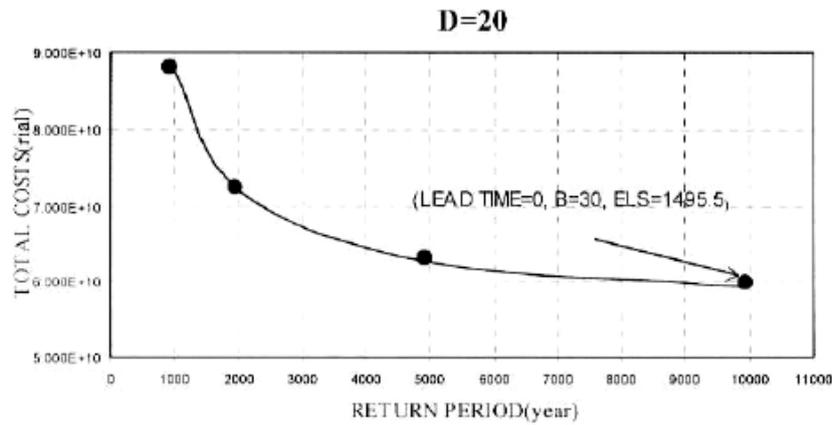
Under these assumptions, optimal risk analysis was conducted for each damage coefficient. Detailed analyses on how early warning time changes with the damage coefficient and design flood return period are provided in the reference. In this article, due to page limitations, we present the final results (Figures 3, 4, 5).



**Figure 3:** Cost function according to flood return period with damage factor of 4



**Figure 4:** Cost function according to flood return period with damage factor of 10



**Figure 5:** Cost function according to flood return period with damage factor of 20

It's evident that with an increase in the flood return period, the optimal pre-warning time also increases. For instance, for a damage factor of 4, the optimal pre-warning value changes from zero for a 1000-year flood to 108 hours for a 10,000-year flood. This trend is attributed to the higher risk of floods with lower return periods. Additionally, as the damage coefficient ( $D$ ) increases, the steepness of the false alarm probability function compared to the pre-warning time (5% error for every 12 hours of pre-warning time) leads to a decrease in the optimal pre-warning time for the final options after risk analysis. It decreases from 108 hours for a damage factor of 4 to zero for a damage factor of 20 (Table 2).

**Table 2:** Changes in the optimal pre-warning time according to the damage coefficient after risk analysis

<i>Damage coefficient (D)</i>	<i>Optimal pre-warning time (hr)</i>
4	108
10	36
20	0

## 7. Summary and Conclusion

In this study, we conducted a review of flood warning systems, with a particular focus on automatic flood warning systems. We explained two approaches for selecting flood designs: the use of criteria and risk analysis. Subsequently, a novel method for spillway design, considering the possibility of utilizing flood warning systems and selecting the best option through risk analysis, was presented.

Continuing the work, due to the substantial number of calculations required for the design, we developed software designed to facilitate the spillway design process. Following this, the initial selection of a dam from those under design was made using this software. The results, along with sensitivities to advance warning time, damage coefficient, and various return periods, were presented. As expected, an increase in the return period led to the addition of the optimal early warning time, while an increase in the downstream damage coefficient resulted in a reduction.

In conclusion, this study highlights that equipping watershed with flood control systems not only reduces potential flood risks for residents and facilities in floodplains but also allows for a reduction in the costs of constructing facilities such as dams in these basins.

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